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FILTRATION–DIFFUSION HEAT AND MASS TRANSFER IN POROUS MEDIUM

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Abstract—Luikov and Mikhailov's mass- and heat-transfer equations for the porous bodies have been modified to include the molar (hydrodynamic) heat and mass transfer. A problem dealing with the combined molar and molecular heat and mass transfer in a semi-infinite cylindrical porous body under boundary conditions of the first kind (constant heat- and mass-transfer potentials on the surface) has been studied neglecting heat conduction and mass diffusion in the axial direction. The equations have been solved by the use of Laplace and Hankel transforms.

NOMENCLATURE

- t, temperature [°C];
- u, moisture content [g moisture/g dry matter];
- r, radial co-ordinate [cm];
- R, bounding surface radius [cm];
- z, axial co-ordinate [cm];
- τ, time [s];
- λ, thermal conductivity coefficient [cal/cm s degC];
- C, specific heat capacity of moist body [cal/g degC];
- γ_0, density of absolutely dry matter [g/cm³];
- a, thermal diffusivity coefficient [cm²/s];
- a', moisture conductivity coefficient [cm²/s];
- ρ, specific heat of evaporation [cal/g];
- δ, thermal gradient coefficient [1/degC];
- ϵ, coefficient of moisture internal evaporation;
- V_z, uniform average velocity of the moisture (liquid or vapour) in the capillaries of the cylinder in the direction of z decreasing;
- $J_n(x)$, Bessel function of the nth order and of first kind.

1. INTRODUCTION

LUIKOV AND MIKHAILOV [1] have considered the following system of equations of heat and mass transfer for analysing various problems in drying processes under different boundary conditions.

$$\frac{\partial t}{\partial \tau} = a\,\nabla^2 t + (\epsilon\rho/c)\frac{\partial u}{\partial \tau} \tag{1}$$

$$\frac{\partial u}{\partial \tau} = a'\nabla^2 u + a'\delta\nabla^2 t \tag{2}$$

The above equations have been derived on the assumption of very slow diffusive motion of moisture in pores and capillaries in the body. The effect of this motion has therefore been neglected in comparison with the heat conduction and mass diffusion effects in the above derivation. However, if the capillary motion of the moisture in the body is also taken into account the above equations are modified as (6) and (7) below. With this modification we discuss a problem dealing with the simultaneous heat and mass transfer for a semi-infinite porous circular cylinder, under boundary conditions of the first kind neglecting the mass diffusion and heat conduction effects in the axial direction and considering that the moisture (liquid or vapour) moves in the capillaries with a constant average velocity V_z in the direction of z (current axial co-ordinate) decreasing.

2. BASIC EQUATIONS

The equation of Luikov and Mikhailov for mass diffusion in a porous medium [equation (2)] can be written as

$$\gamma_0\frac{\partial u}{\partial \tau} = -\operatorname{div} J_{\text{dif}} \tag{3}$$

Where

$$J_{\text{dif}} = -a'\gamma_0 \nabla u - a'\gamma_0 \delta \nabla t \qquad (4)$$

The above equation holds on the assumption of very slow motion of moisture in pores and capillaries in the body. If in body capillaries and pores, not only mass diffusion but also hydrodynamic motion of moisture (liquid or vapour) occurs at some average velocity v (v = const.) then differential mass-transfer equation will be as follows:

$$\gamma_0 \frac{\partial u}{\partial \tau} = -\operatorname{div}(J_{\text{dif}} + \gamma_0 u v) = -v\gamma_0 \operatorname{grad} u - \operatorname{div} J_{\text{dif}}$$

or

$$\gamma_0 \frac{\partial u}{\partial \tau} + v\gamma_0 \operatorname{grad} u = -\operatorname{div} J_{\text{dif}} \qquad (5)$$

$$\frac{\mathrm{D}u}{\mathrm{D}\tau} = a'\nabla^2 u + a'\delta\nabla^2 t \qquad (6)$$

where $(\mathrm{D}u/\mathrm{D}\tau) = (\partial u/\partial \tau) + v \operatorname{grad} u$, i.e. the substantial derivative of u.

In the same way we get the heat-transfer equation

$$\frac{\mathrm{D}t}{\mathrm{D}\tau} = a\nabla^2 t + (\epsilon\rho/c)\frac{\partial u}{\partial \tau}\dagger \qquad (7)$$

† The above equations (6) and (7) are similar to those of Luikov and Mikhailov describing heat and mass transfer in moving solutions. These equations are

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = a\nabla^2 t + \sigma \mathrm{D} k_1 \nabla^2 \rho_{10} \qquad (a)$$

$$\frac{\mathrm{d}\rho_{10}}{\mathrm{d}\tau} = \mathrm{D}\nabla^2 \rho_{10} + \sigma \mathrm{D} \nabla^2 t \qquad (b)$$

where $\mathrm{d}/\mathrm{d}\tau = \partial/\partial\tau + v$ grad.

The above equations can be written as

$$\frac{\mathrm{d}t}{\mathrm{d}\tau} = (a - \sigma^2 k_1 \mathrm{D})\nabla^2 t + \sigma k_1 \frac{\mathrm{d}\rho_{10}}{\mathrm{d}\tau} \qquad (c)$$

$$\frac{\mathrm{d}\rho_{10}}{\mathrm{d}\tau} = \mathrm{D}\nabla^2 \rho_{10} + \sigma \mathrm{D} \nabla^2 t \qquad (d)$$

In this form the equations describe the heat and mass transfer in binary gas mixtures without sources ($I_i = 0$). Reference A. V. LUIKOV and Y. A. MIKHAILOV, *Theory of Energy and Mass Transfer* (Russian), 2nd ed., p. 48 (1963) and equations (2.1.86) and (2.1.87). It is seen that the only difference between the equations (c) and (d) and (6) and (7) is the presence of a total derivative in the last term on the right-hand side of (c) instead of partial derivative occurring in (7).

3. THE PROBLEM

The problem to be treated here may be stated as follows:

Moisture (liquid or vapour) is filtrated through a porous circular cylinder under the action of hydrostatic pressure transporting along the cylinder-axis. Filtration (molar) transfer of moisture and heat prevail over molecular transfer (by mass diffusion and heat conduction) in the axial direction. The velocity of molar transfer is assumed constant and equal to V_z (constant). Determine non-stationary fields of moisture content and temperature.

Mathematically:

$$\frac{\partial u}{\partial \tau} + V_z \frac{\partial u}{\partial z} = a'\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) + a'\delta\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) \qquad (8)$$

$$\frac{\partial t}{\partial \tau} + V_z \frac{\partial t}{\partial z} = a\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) + (\epsilon\rho/c)\frac{\partial u}{\partial \tau} \qquad (9)$$

$$0 < r < R \quad t > 0 \quad z > 0$$

Initial conditions

$$\left.\begin{aligned} u(r, z, 0) &= u_0 \\ t(r, z, 0) &= t_0 \end{aligned}\right\} \qquad (10)$$

Boundary conditions

$$\left.\begin{aligned} u(R, z, \tau) &= u_1 \\ t(R, z, \tau) &= t_1 \end{aligned}\right\} \qquad (11)$$

$$\left.\begin{aligned} u(r, 0, \tau) &= u_0 \\ t(r, 0, \tau) &= t_0 \end{aligned}\right\} \qquad (12)$$

4. SOLUTION OF THE PROBLEM

The finite Hankel transform of a function $u(r, z, \tau)$ is given by $u^*(s, z, \tau)$, Sneddon [2]

$$u^*(s, z, \tau) = \int_0^R u(r, z, \tau)\, r J_0(rs)\, \mathrm{d}r \qquad (13)$$

where J_0 is the Bessel function of first kind and of zero order and s is the root of the characteristic equation

$$J_0(Rs) = 0 \qquad (14)$$

Multiplying the equations (8) and (9) throughout by $rJ_0(rs)$ and integrating with respect to r taking into consideration the conditions (11), the above equations give

$$\frac{\partial u^*}{\partial \tau} + V_z \frac{\partial u^*}{\partial z} = a' \, Rs \, J_1(Rs) \, u_1 - a' s^2 u^* + a' \delta \, Rs \, J_1(Rs) \, t_1 - a' \delta \, s^2 t^* \quad (15)$$

$$\frac{\partial t^*}{\partial \tau} + V_z \frac{\partial t^*}{\partial z} = a \, Rs \, J_1(Rs) \, t_1 - a s^2 t^* + (\epsilon \rho / c) \frac{\partial u^*}{\partial \tau} \quad (16)$$

The above equations are to be solved subject to the following conditions which have been obtained by applying the finite Hankel transform to the initial conditions (10) and boundary conditions (12).

$$\left. \begin{aligned} u^* &= u_0^* = u_0(R/s) J_1(Rs) \\ t^* &= t_0^* = t_1(R/s) J_1(Rs) \end{aligned} \right\} \begin{aligned} \tau &= 0 \\ z &> 0 \end{aligned} \quad (17)$$

$$\left. \begin{aligned} u^* &= u_0^* = u_0(R/s) J_1(Rs) \\ t^* &= t_0^* = t_0(R/s) J_1(Rs) \end{aligned} \right\} \begin{aligned} z &= 0 \\ \tau &> 0 \end{aligned} \quad (18)$$

In this problem it has been found convenient to apply the Laplace transform with respect to z instead of τ. We define the Laplace transform of $f(r, z, \tau)$ as $\bar{f}(r, p', \tau)$ where

$$\bar{f}(r, p', \tau) = \int_0^\infty f(r, z, \tau) \exp[-p' z] \, dz \quad (19)$$

Applying the above transform to equation (15) and (16), we get

$$\frac{d\overline{u^*}}{d\tau} + V_z(p'\overline{u^*} - u_0 (R/s) J_1(Rs)) = a' \, Rs \, J_1(Rs) \, (u_1/p') - a' s^2 \overline{u^*} + a' \delta \, Rs \, J_1(Rs) \, (t_1/p') - a' \delta \, s^2 \overline{t^*} \quad (20)$$

$$\frac{d\overline{t^*}}{d\tau} + V_z \, [p'\overline{t^*} - t_0 (R/s) J_1(Rs)] = a \, Rs \, J_1 (Rs) \, (t_1/p') - a s^2 \overline{t^*} + (\epsilon \rho / c) \, (d\overline{u^*}/d\tau) \quad (21)$$

with initial conditions

$$\left. \begin{aligned} \overline{u^*} &= \frac{u_0}{p'} \frac{R}{s} J_1(Rs) \\ \overline{t^*} &= \frac{t_0}{p'} \frac{R}{s} J_1(Rs) \end{aligned} \right\} \quad (22)$$

Eliminating $\overline{t^*}$ between simultaneous differential equations (20) and (21) we get a single second order equation

$$\frac{d^2 \overline{u^*}}{d\tau^2} + \left(2 p' V_z + (a + a') s^2 + \frac{\epsilon \rho}{c} a' \delta s^2\right) \frac{d\overline{u^*}}{d\tau} + (p' V_z + a' s^2)(p' V_z + a s^2)] \, \overline{u^*}$$
$$= p' V_z^2 u_0 (R/s) J_1(Rs) + a a' R s^3 J_1(Rs)(u_1/p') + V_z a' Rs \, J_1(Rs) \, u_1 + V_z a' \delta \, Rs \, J_1(Rs)(t_1 - t_0) \quad (23)$$

The solution of this equation admissible to the problem and satisfying the transformed initial conditions (22) is given below:

$$\overline{u^*} = A - \frac{N}{p'} \frac{\exp[-H\tau - p' V_z \tau - (\tau/2) \sqrt{(A' + B'p')}]}{\sqrt{(A' + B'p')}}$$
$$- A \, [-p'V_z - H + \tfrac{1}{2} \sqrt{(A' + B'p')}] \frac{\exp[-H\tau - p' V_z \tau - (\tau/2) \sqrt{(A' + B'p')}]}{\sqrt{(A' + B'p')}}$$
$$+ \frac{u_0}{p'} \frac{R}{s} J_1(Rs) \, [-p'V_z - H + \tfrac{1}{2} \sqrt{(A' + B'p')}] \frac{\exp[-H\tau - p' V_z \tau - (\tau/2) \sqrt{(A' + B'p')}]}{\sqrt{(A' + B'p')}} \quad (24)$$

where

$$A = \frac{A_1 p'^2 + A_2 p' + A_3}{p'(B_1 p'^2 + B_2 p' + B_3)} \quad (25)$$

$$A_1 = V_z^2 \, u_0 \, (R/s) \, J_1(Rs) \quad (26)$$

$$A_2 = V_z \, [a' Rs \, J_1 (Rs) \, u_1 + a' \delta \, Rs \, J_1(Rs) \, (t_1 - t_0) + a Rs \, J_1(Rs) \, u_0] \quad (27)$$

$$A_3 = aa' Rs^3 J_1(Rs)\, u_1 \tag{28}$$

$$\left.\begin{aligned} B_1 &= V_z^2 \\ B_2 &= V_z\,(a + a')s^2 \\ B_3 &= aa'\, s^4 \end{aligned}\right\} \tag{29}$$

$$N = a' Rs J_1(Rs)\,(u_1 - u_0) + a'\delta\, Rs J_1(Rs)\,(t_1 - t_0) \tag{30}$$

$$A' = s^4 [(a - a')^2 + 2a'\delta\,(\epsilon\rho/c)\,(a + a') + (\epsilon^2\rho^2/c^2)\, a'^2\, \delta^2] \tag{31}$$

$$B' = 4V_z\, a'\, \delta\, s^2\,(\epsilon\rho/c) \tag{32}$$

also

$$H = \tfrac{1}{2}\,(a + a')s^2 + (\tfrac{1}{2}\,\epsilon\rho/c)\, a'\delta s^2 \tag{33}$$

These substitutions have been effected for ease of applying the inversion theorem with respect to z, which is

$$f(r, z, \tau) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \bar{f}(r, p', \tau)\exp[-p'z]\, dp' \tag{34}$$

The inversion of the various terms in the right-hand side of equation (24) with respect to z is quite cumbersome and we refrain from explaining the same in detail for the sake of brevity. It can be noted, however, that the inversion in respect of all the terms on the right-hand side of equation (24) excepting the first, has been effected by application of the convolution theorem Churchill [3]. As a result of the inversion with respect to z, it is found that the solution breaks into two parts, one valid for the region $z < V_z\tau$ and the other for the region $z > V_z\tau$

$$u^*(s, z, \tau) = \frac{1}{V_z^2}\left[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)}\exp[\alpha z] + \frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)}\exp[\beta z]\right] \quad z < h \tag{35}$$

where

$$\alpha = -\frac{a's^2}{V_z}$$

$$\beta = -\frac{as^2}{V_z}$$

$$h = V_z\tau$$

and

$$\left.\begin{aligned} u^*(s, z, \tau) = {} & \frac{1}{V_z^2}\left[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)}\exp[\alpha z] + \frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)}\exp[\beta z]\right] + \\ & \frac{1}{V_z}\exp[-H\tau]\int_h^z \left\{\left[\begin{aligned} & \frac{[A_2 - (A_1 B_2/B_1)]\,\alpha + [A_3 - (A_1 B_3/B_1)]}{(\alpha - \beta)}\exp[\alpha\lambda] \\ & + \frac{[A_2 - (A_1 B_2/B_1)]\,\beta + [A_3 - (A_1 B_3/B_1)]}{(\beta - \alpha)}\exp[\beta\lambda] \end{aligned}\right]\times\right. \\ & \left.\frac{\exp\{[-(A'/B')\,(z - \lambda - h)] - [\tau^2 B'/16(z - \lambda - h)]\}}{\sqrt{[\pi B'(z - \lambda - h)]}}\right\} d\lambda + \\ & \frac{H}{V_z^2}\exp[-H\tau]\int_h^z \left\{\left[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)}\exp[\alpha\lambda] + \frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)}\exp[\beta\lambda]\right]\times\right. \end{aligned}\right\} \tag{36}$$

$$\left.\frac{\exp\{-(A'/B')(z-\lambda-h)-[\tau^2B'/16(z-\lambda-h)]\}}{\sqrt{[\pi B'(z-\lambda-h)]}}\right\}d\lambda+\tfrac{1}{2}u_0(R/s)J_1(Rs)\}\times \tag{36}$$

$$\exp[-H\tau]\int_0^{z-h}\frac{\tau B'}{4\lambda}\cdot\frac{\exp[-(A'/B')\lambda-(\tau^2B'/16\lambda)]}{\sqrt{(\pi B'\lambda)}}d\lambda-1/2V_z^2\times$$

$$\exp[-H\tau]\int_h^z\left\{\left[\frac{A_3}{\alpha\beta}+\frac{A_1\alpha^2+A_2\alpha+A_3}{\alpha(\alpha-\beta)}\exp[\alpha\lambda]+\frac{A_1\beta^2+A_2\beta+A_3}{\beta(\beta-\alpha)}\exp[\beta\lambda]\right]\times\right.$$

$$\left.\frac{\tau B'}{4(z-\lambda-h)}\cdot\frac{\exp\{-(A'/B')(z-\lambda-h)-[\tau^2B'/16(z-\lambda-h)]\}}{\sqrt{[\pi B'(z-\lambda-h)]}}\right\}d\lambda-$$

$$[Hu_0(R/s)J_1(Rs)+N]\exp[-H\tau]\int_0^{z-h}\frac{\exp[-A'/B'\lambda-\tau^2B'/16\lambda]}{\sqrt{(\pi B'\lambda)}}d\lambda$$

$$z>h$$

$u(r, z, \tau)$ can now be obtained from $u^*(s, z, \tau)$ by the application of the inversion theorem for finite Hankel transform stated as

$$f(r,z,\tau)=\frac{2}{R^2}\sum_{s_i}f^*(s,z,\tau)\frac{J_0(rs_i)}{J_1^2(Rs_i)} \tag{37}$$

where s_i are the roots of the equation (14). Eliminating $\overline{u^*}$ from equation (24) with the help of differential equation (20) and proceeding for $\overline{t^*}$ exactly as in the case of $\overline{u^*}$, we get on application of the Laplace inversion theorem with respect to z, two solutions valid for two regions $z < h$ and $z > h$ as below

$$-a'\delta s^2t^*=\frac{1}{V_z}\left[\frac{C_1\alpha+C_2}{(\alpha-\beta)}\exp[\alpha z]+\frac{C_1\beta+C_2}{(\beta-\alpha)}\exp[\beta z]\right]+$$

$$\frac{a's^2}{V_z^2}\left[\frac{A_3}{\alpha\beta}+\frac{A_1\alpha^2+A_2\alpha+A_3}{\alpha(\alpha-\beta)}\exp[\alpha z]+\frac{A_1\beta^2+A_2\beta+A_3}{\beta(\beta-\alpha)}\exp[\beta z]\right]$$

$$-a'RsJ_1(Rs)u_1-a'\delta RsJ_1(Rs)t_1\quad z<h \tag{38}$$

where

$$C_1=V_z[a'RsJ_1(Rs)(u_1-u_0)+a'\delta RsJ_1(Rs)(t_1-t_0)] \tag{39}$$

$$C_2=aa'Rs^3J_1(Rs)(u_1-u_0) \tag{40}$$

and

$$-a'\delta s^2t^*=\left[HN+\frac{A_1}{B_1}\left(H^2-\frac{A'}{4}\right)-\frac{A_1}{B_1}a's^2H-a's^2N\right]\exp[-H\tau]\times$$

$$\int_0^{z-h}\frac{\exp[-(A'/B')\lambda-(\tau^2B'/16\lambda)]}{\sqrt{(\pi B'\lambda)}}d\lambda+\frac{1}{2}\left(a's^2\frac{A_1}{B_1}+N\right)\exp[-H\tau]\times$$

$$\int_0^{z-h}\frac{\tau B'}{4\lambda}\cdot\frac{\exp[-(A'\lambda/B')-(\tau^2B'/16\lambda)]}{\sqrt{(\pi B'\lambda)}}d\lambda+\frac{1}{V_z^2}\left(a's^2V_z+\frac{B'}{4}-HV_z\right)\exp[-H\tau]\times \tag{41}$$

$$
\left.
\begin{aligned}
&\int_h^z \Bigg\{ \Bigg[\frac{[A_2 - (A_1B_2/B_1)]\,\alpha + [A_3 - (A_1B_3/B_1)]}{(\alpha - \beta)} \exp[\alpha\lambda] + \frac{[A_2 - (A_1B_2/B_1)]\,\beta + [A_3 - (A_1B_3/B_1)]}{(\beta - \alpha)} \\
&\exp[\beta\lambda] \Bigg] \times \frac{\exp\{-(A'/B')(z - \lambda - h) - [\tau^2 B'/16(z - \lambda - h)]\}}{\sqrt{[\pi B'(z - \lambda - h)]}} \Bigg\} \,\mathrm{d}\lambda + \\
&\frac{1}{V_z^2}\left(a's^2 H - H^2 + \frac{A'}{4}\right) \exp[-H\tau] \int_h^z \Bigg\{ \Bigg[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)} \exp[\alpha\lambda] + \\
&\frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)} \exp[\beta\lambda] \Bigg] \times \frac{\exp\{-(A'/B')(z - \lambda - h) - [\tau^2 B'/16(z - \lambda - h)]\}}{\sqrt{[\pi B'(z - \lambda - h)]}} \Bigg\} \,\mathrm{d}\lambda - \\
&\frac{1}{2V_z} \exp[-H\tau] \int_h^z \Bigg\{ \Bigg[\frac{[A_2 - (A_1B_2/B_1)]\,\alpha + [A_3 - (A_1B_3/B_1)]}{(\alpha - \beta)} \exp[\alpha\lambda] + \\
&\frac{[A_2 - (A_1B_2/B_1)]\,\beta + [A_3 - (A_1B_3/B_1)]}{(\beta - \alpha)} \exp[\beta\lambda] \Bigg] \times \frac{\tau B'}{4(z - \lambda - h)} \cdot \\
&\frac{\exp\{-(A'/B')(z - \lambda - h) - [\tau^2 B/16(z - \lambda - h)]\}}{\sqrt{[\pi B'(z - \lambda - h)]}} \Bigg\} \,\mathrm{d}\lambda + \frac{a's^2}{V_z^2} \Bigg[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)} \\
&\exp[\alpha z] + \frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)} \exp[\beta z] \Bigg] - \frac{a's^2}{2V_z^2} \exp[-H\tau] \times \\
&\int_h^z \Bigg\{ \Bigg[\frac{A_3}{\alpha\beta} + \frac{A_1\alpha^2 + A_2\alpha + A_3}{\alpha(\alpha - \beta)} \exp[\alpha\lambda] + \frac{A_1\beta^2 + A_2\beta + A_3}{\beta(\beta - \alpha)} \exp[\beta\lambda] \Bigg] \times \\
&\frac{\tau B'}{4(z - \lambda - h)} \cdot \frac{\exp\{-(A'/B')(z - \lambda - h) - [\tau^2 B'/16(z - \lambda - h)]\}}{\sqrt{[\pi B'(z - \lambda - h)]}} \Bigg\} \,\mathrm{d}\lambda + \\
&\frac{1}{V_z} \Bigg[\frac{[A_2 - (A_1B_2/B_1)]\,\alpha + [A_3 - (A_1B_3/B_1)]}{(\alpha - \beta)} \exp[\alpha z] + \\
&\frac{[A_2 - (A_1B_2/B_1)]\,\beta + [A_3 - (A_1B_3/B_1)]}{(\beta - \alpha)} \exp[\beta z] \Bigg] - a'Rs\,J_1(Rs)u_1 - a'\delta Rs J_1(Rs)t_1. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad z > h
\end{aligned}
\right\} \quad (41)
$$

The application of Hankel inversion theorem (37) to $t^*(s, z, \tau)$ gives

$$
t(r, z, \tau) = \frac{2}{R^2} \sum_{s_i} t^*(s, z, \tau) \frac{J_0(rs_i)}{J_1^2(Rs_i)} \qquad (42)
$$

5. RESULTS

In this paper the equations governing the combined heat and mass diffusion in a porous medium have been considered taking into account the molar heat and mass transfer due to the motion of the moisture in the capillaries of the porous body at a constant velocity. A problem concerning the heat and mass transfer in a semi-infinite porous circular cylinder has been studied under the boundary conditions of the first kind, neglecting the axial conduction considering that moisture moves in the capillaries at a constant velocity V_z. As should be expected it comes out that the solution breaks into two, one applicable to each of the regions $z < h$ and the other for $z > h$ ($h = V_z \tau$). The region of interest from a practical point of view is one for which z is large. For this region the finite integrals occurring in the equations (36) and (41) can be easily approximated.

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Résumé—Les équations de transport de chaleur et de masse de Luikov et Mikhailov pour les corps poreux ont été modifiées pour y inclure le transport de chaleur et de masse molaire (hydrodynamique). Un problème traitant du transport de chaleur et de masse combiné, molaire et moléculaire, dans un corps poreux cylindrique semi-infini avec des conditions aux limites de première espèce (potentiels de transport de chaleur et de masse constants sur la surface) a été étudié en négligeant la conduction de la chaleur et la diffusion de la masse dans la direction axiale. Les équations ont été résolues à l'aide des transformées de Laplace et de Hankel.

Zusammenfassung—Luikov's und Mikhailov's Stoff- und Wärmeübertragungsgleichungen für poröse Körper wurden abgewandelt, um den molaren (hydrodynamischen) Wärme- und Stoffübergang mit einzuschliessen. Unter Vernachlässigung der Wärmeleitung und der Stoffdiffusion in axialer Richtung wurde ein Problem untersucht, das sich mit der Kombinierten der molaren und molekularen Wärme- und Stoffübertragung in einem halbunendliehen sylindrischen porösen Körper bei Grensschichtbedingungen erster Art (konstantes Wärme- und Stoffübertragungspotential an der Oberfläche) befasst. Die Gleichungen wurden durch Verwendung von Laplace und Hankeltransformationen gelöst.

Аннотация—Уравнения тепло-и массопереноса в пористых телах, выведенные Лыковым и Михайловым, преобразованы с учетом молярного (гидродинамического) переноса тепла и вещества. Рассмотрена задача о сложном молярном и молекулярном переносе тепла и вещества в полубесконечном пористом цилиндре при граничных условиях первого рода (постоянные потенциалы тепло-и массопереноса на поверхности) без учета теплопроводности и диффузии вещества в осевом направлении. Уравнения решены с помощью преобразований Лапласа и Ханкеля.